

Equivalent spin-orbit interaction in two-polariton Jaynes-Cummings-Hubbard model

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A hybrid quantum system combines two or more distinct quantum components, exhibiting features not seen in these individual systems. In this work, we study the one-dimensional Jaynes-Cummings-Hubbard model in the two-excitation subspace. We find that the center momentum of two-excitation induces a magnetic flux piercing the 4-leg ladder in the auxiliary space. Furthermore, it is shown that the system in π -center-momentum subspace is equivalent to a chain system for spin-1 particle with spin-orbit coupling. As a simple application, based on this concise description, a series of bound-pair eigenstates is presented, which displays long-range correlation.

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I. INTRODUCTION

A hybrid quantum system combines two or more distinct quantum components, exhibiting features not seen in these individual systems. This provides a promising platform to study novel quantum phenomena. The Jaynes-Cummings-Hubbard (JCH) model is an archetype of such hybridization, which consists of the JCH and the coupled cavities. This model was proposed for the use of the atom-light interaction in coupled microcavity arrays to create strongly correlated many-body models [1–3]. It has received intensive study in a variety of directions (See the review [4, 5] and references therein).

The researches mainly focus on the ground state phase of many-particle system and the dynamics in single-particle system. Mott insulator phase and superfluid phase are identified by the traditional order parameter, the average of the annihilation operator [3] and the observable quantities, atomic concurrence and photon visibility [6]. The single-particle dynamics suggests that this hybrid architecture can be the quantum coherent device to transfer and store quantum information as well as to create the laser-like output [7–9]. Recently, few-body problem for the JCH Hamiltonian is also investigated [10, 11], claiming the existence of two-polariton bound states when the photon-atom interaction is sufficiently strong.

In this work, we study the one-dimensional JCH model in the two-excitation subspace. In each invariant subspace, the sub-Hamiltonian is equivalent to a 4-leg ladder with an effective flux, which is proportional to the center momentum of two excitations. It is shown that in π -center-momentum subspace, the ladder system can be reduced to a chain system of spin-1 particle with spin-orbit coupling. As a simple application, based on this concise description, a series of bound-pair eigenstates is presented, which display long-range polaritonic entanglement.

This paper is organized as follows. In Section II, we present the JCH model and the basis set. In Section III, the equivalent Hamiltonian is given. In Section IV, in π -center-momentum subspace, the equivalent Hamiltonian is reduced to a chain system of spin-1 particle with spin-orbit coupling. In Section V, a series of bound-pair eigenstates is constructed. In Section VI, we investigate the quantum correlation for the bound-pair states. Finally, we give a summary and discussion in Section VII.

II. JCH MODEL

The Jaynes-Cummings model describes a cavity array doped with a single two-level atoms embedded in each cavity and the dipole interaction leads to dynamics involving photonic and atomic degrees of freedom, which is in contrast to the widely studied Bose-Hubbard model. Such hybrid system can be implemented with the defect array in photonic crystal [12] or Josephson junction array in cavity [7]. The Hamiltonian of a hybrid system, or a lattice atom-photon system,

$$H = H_0 + H_{JC} + H_C \quad (1)$$

can be written as three parts, free Hamiltonians of atom and photon,

$$H_{AP} = \omega_a \sum_{l=1}^N a_l^\dagger a_l + \omega_b \sum_{l=1}^N |e\rangle_l \langle e|, \quad (2)$$

the JC type cavity-atom interaction in the i -th defect

$$H_{JC} = \lambda \sum_{l=1}^N \left(a_l^\dagger |g\rangle_l \langle e| + \text{H.c.} \right), \quad (3)$$

with strength λ and the photon hopping between nearest neighbor cavities

$$H_C = -\kappa \sum_{l=1}^N \left(a_l^\dagger a_{l+1} + \text{H.c.} \right), \quad (4)$$

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with hopping integral constant κ for the tunneling between adjacent cavities. Here, $|g\rangle_i$ ($|e\rangle_i$) denotes the ground (excited) state of the atom placed at i th cavity; a_i^\dagger and a_i are the creation and annihilation operators of a photon at defect i . Obviously the total excitation number

$$\hat{\mathcal{N}} = \sum_{i=1} \hat{\mathcal{N}}_i = \sum_{i=1} \left(a_i^\dagger a_i + \sigma_i^z + \frac{1}{2} \right), \quad (5)$$

is conserved quantity for the Hamiltonian H , i.e., $[H, \hat{\mathcal{N}}] = 0$, where $\sigma_i^z |e\rangle_l = |e\rangle_l$ and $\sigma_i^z |g\rangle_l = -|g\rangle_l$. It can be seen that $\hat{\mathcal{N}}$ is just the single excitation number of the polaritons. For each cavity, the basis state can be expressed as $\{|n\rangle_l |e\rangle_l, |n\rangle_l |g\rangle_l\}$, where the basis state of the Fock space for l -th cavity is $|n\rangle_l = \left(a_l^\dagger \right)^n / \sqrt{n!} |0\rangle_l$. In this paper, we consider the invariant subspace with $\mathcal{N} = 2$, which is spanned by the basis in the form

$$\begin{pmatrix} |2\rangle_i \\ |1\rangle_i |1\rangle_{i+j} \\ |e\rangle_i |e\rangle_{i+j} \\ |e\rangle_i |1\rangle_{i'} \end{pmatrix} \equiv \begin{pmatrix} |2\rangle_i \langle 0| \\ |1\rangle_i |1\rangle_{i+j} \langle 0|_i \langle 0| \\ |e\rangle_i |e\rangle_{i+j} \langle g|_i \langle g| \\ |e\rangle_i |1\rangle_{i'} \langle 0|_i \langle g| \end{pmatrix} |G\rangle, \quad (6)$$

where $j \geq 1$ and $|G\rangle \equiv \prod_{i=1} |g\rangle_i |0\rangle_i$ denotes the empty state with zero \mathcal{N} . We denote the matrix representation of the Hamiltonian of Eq. (1) in the basis of Eq. (6) as \underline{H} . In the case of real values of κ and λ , we have $\underline{H}^* = \underline{H}$, which indicates that \underline{H} has time-reversal symmetry.

III. 4-LEG LADDER WITH FLUX

The system is translational invariant [13]. In the two-particle Hilbert space, the Hamiltonian H can be written as $H = \sum_k H_k$, where

$$H_k = \sum_{j=1}^4 \sum_{l=1}^4 (J_l |j, l, k\rangle \langle j+1, l, k| + \lambda |j, l, k\rangle \langle j, l+1, k| + \text{H.c.}) + \sum_{j=1}^4 \sum_{l=1}^4 (\mu_l |j, l, k\rangle \langle j, l, k|) + h_k, \quad (7)$$

and

$$h_k = \sum_{j=0, l=1, 3} J_l |j, l, k\rangle \langle j+1, l, k| + \sqrt{2}\lambda |0, 1, k\rangle \langle 0, 2, k| + \sqrt{2}J_2 |0, 2, k\rangle \langle 1, 2, k| + \sum_{j=0, l=1, 2} \mu_l |j, l, k\rangle \langle j, l, k| + \text{H.c.} \quad (8)$$

where we have taken $|j, 5, k\rangle \equiv |j, 1, k\rangle$ for $j \geq 1$, and $|0, 1, k\rangle \equiv |0, 3, k\rangle$. The parameters reads

$$J_{1,2,3,4} = \left(-\kappa e^{ik/2}, -2\kappa \cos(k/2), -\kappa e^{-ik/2}, 0 \right), \quad (9)$$

and

$$\mu_{1,2,3,4} = (\omega_a + \omega_b, 2\omega_a, \omega_a + \omega_b, 2\omega_b). \quad (10)$$

Here the set of states $\{|j, l, k\rangle\}$ is defined as following: For $j \geq 1$, it reads

$$\begin{pmatrix} |j, 1, k\rangle \\ |j, 2, k\rangle \\ |j, 3, k\rangle \\ |j, 4, k\rangle \end{pmatrix} = \sum_l \frac{e^{ik(l+j/2)}}{\sqrt{N}} \begin{pmatrix} |1\rangle_l |e\rangle_{l+j} \\ |1\rangle_l |1\rangle_{l+j} \\ |e\rangle_l |1\rangle_{l+j} \\ |e\rangle_l |e\rangle_{l+j} \end{pmatrix}, \quad (11)$$

and

$$\begin{pmatrix} |0, 1, k\rangle \\ |0, 2, k\rangle \end{pmatrix} = \sum_l \frac{e^{ikl}}{\sqrt{N}} \begin{pmatrix} |1\rangle_l |e\rangle_l \\ |2\rangle_l \end{pmatrix}, \quad (12)$$

The expression of H_k in Eq. (7) has a clear physical meaning: $|j, l, k\rangle$ denotes the site state for j th site on the l leg of a 4-leg ladder system with the effective magnetic piercing the plaquette. The flux is proportional to the center momentum of two excitations. The structure of H_k is schematically illustrated in Fig. 1. We note that the matrix representation of H_k in the basis of Eqs. (11) and (12), \underline{H}_k breaks the time-reversal symmetry. Nevertheless, we still have $\sum_k \underline{H}_k = \sum_k \underline{H}_k^*$ due to the fact that $\underline{H}_k^* = \underline{H}_{-k} = \underline{H}_{4\pi-k}$. In essence, the nonzero plaquette flux arises from the relation between the complex coupling constants $J_1 = J_3^* = -\kappa e^{ik/2}$. In contrast, one can see from H_k that the complex λ cannot induce a nonzero plaquette flux. We would like to point that the effective magnetic field in the present model is intrinsic, different from that discussed in Refs. [14] and [15].

In order to understand the mechanism of the effective flux, we investigate the exchange process between photon and atomic excitations from the state $|\psi(l, l+j)\rangle$ to state $|\psi(l+j, l)\rangle$, where

$$|\psi(l, l')\rangle = (|1\rangle_l |e\rangle_{l'} - |1\rangle_{l+1} |e\rangle_{l'+1}) / \sqrt{2}. \quad (13)$$

The action of H provides at least two paths for this task: The first one is described as

$$\begin{aligned} \text{I: } & |\psi(l, l+j)\rangle \\ & \rightarrow (|1\rangle_l |1\rangle_{l+j} - |1\rangle_{l+1} |1\rangle_{l+j+1}) / \sqrt{2} \\ & \rightarrow |\psi(l+j, l)\rangle, \end{aligned} \quad (14)$$

and another one is

$$\begin{aligned} \text{II: } & |\psi(l, l+j)\rangle \\ & \rightarrow (|1\rangle_{l+1} |e\rangle_{l+j} - |1\rangle_l |e\rangle_{l+j+1}) / \sqrt{2} \\ & \rightarrow (|1\rangle_{l+1} |1\rangle_{l+j} - |1\rangle_l |1\rangle_{l+j+1}) / \sqrt{2} \\ & \rightarrow (|1\rangle_{l+1} |1\rangle_{l+j+1} - |1\rangle_l |1\rangle_{l+j}) / \sqrt{2} \\ & \rightarrow e^{i\pi} |\psi(l+j, l)\rangle. \end{aligned} \quad (15)$$

It shows that the exchange process acquires a phase π along the path II, which is equivalent to the effect of a

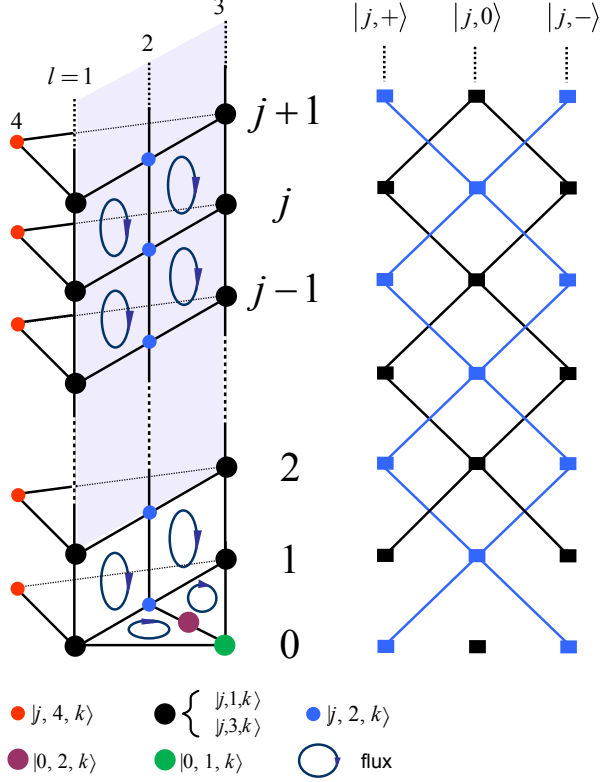


FIG. 1. (Color online) Schematic of the structures of equivalent Hamiltonians for the one-dimensional JC-Hubbard model with two polaritons. (a) In the invariant subspace with center momentum k , the equivalent Hamiltonian H_k describes a four-leg ladder with k -dependent flux. The shadow indicates the semi-infinite uniform ladder. (b) For $k = \pi$, it is equivalent to a spin-1 chain with spin-orbit interaction. The graph of H_π consists of two unconnected subgraphs, characterized by the parity $\Pi = \pm 1$. It indicates that H_π can be further decomposed into two independent parts H_o (blue) and H_e (dark).

flux piercing the loop of two paths. This investigation implies that the origin of the effective magnetic field may be the special statistics property of the atomic excitations: acts as a fermion at the same site but boson for different locations.

Based on the above analysis, the two-polariton scattering problem can be reduced to the study of the single-particle time evolution in the four-leg ladder system. In this paper, we only consider the eigen problem of the two-polariton JC-Hubbard model.

IV. EQUIVALENT HAMILTONIAN IN π -MOMENTUM SUBSPACE

We focus on the case of $k = \pi$ and $\omega_a = \omega_b$, which leads to $H_{AP} = \omega_a \hat{N}$. It is a simple but non-trivial case,

since the hopping along the leg 2 is switched off but the plaquette flux still takes effect. We note that the on-site potentials μ_l of different legs are identical, which allows us to ignore the diagonal terms in H_π .

Introducing the 3-D vector bra and ket for

$$|j\rangle = (|j, +\rangle, |j, 0\rangle, |j, -\rangle), \quad (16)$$

$$\langle j| = \begin{pmatrix} \langle j, +| \\ \langle j, 0| \\ \langle j, -| \end{pmatrix},$$

the Hamiltonian H_π in the π -momentum subspace can be expressed as

$$H_\pi = H_{SO} + 0 \sum_{j=1} |\psi_j\rangle \langle \psi_j|, \quad (17)$$

with $[H_{SO}, \sum_{j=1} |\psi_j\rangle \langle \psi_j|] = 0$, which indicates that H_π is block-diagonal. The sub-Hamiltonian H_{SO} is in the form

$$\begin{aligned} H_{SO} = & \sqrt{2}i\kappa|0\rangle S_x (1 - S_z^2) \langle 1| \\ & i\kappa \sum_{j=1} |j\rangle S_x \langle j+1| + \text{H.c.} \\ & + \sqrt{2}\lambda|0\rangle S_z \langle 0| + 2\lambda \sum_{j=1} |j\rangle S_z \langle j|, \end{aligned} \quad (18)$$

where the Pauli spin matrices for a spin-1 particle are given by

$$S_{x,z} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (19)$$

Here $|j, S_z\rangle$ represents spin-1 particle at j th site with spin polarization $S_z = 0, \pm 1$ and is defined as

$$\begin{cases} |j, \pm\rangle = (|j, 1, \pi\rangle + |j, 3, \pi\rangle \\ \quad \pm |j, 2, \pi\rangle \pm |j, 4, \pi\rangle)/2, \\ |j, 0\rangle = (|j, 1, \pi\rangle - |j, 3, \pi\rangle)/\sqrt{2}, \end{cases} \quad (20)$$

for $j \geq 1$, and

$$|0, \pm\rangle = (|0, 1, \pi\rangle \pm |0, 2, \pi\rangle)/\sqrt{2}. \quad (21)$$

In addition, state $|\psi_j\rangle$ ($j \geq 1$) is defined as

$$\begin{aligned} |\psi_j\rangle = & \frac{1}{\sqrt{2}}(|j, 2, \pi\rangle - |j, 4, \pi\rangle) \\ = & \sum_l \frac{e^{i\pi l}}{\sqrt{2N}} (|1\rangle_l |1\rangle_{l+j} - |e\rangle_l |e\rangle_{l+j}), \end{aligned} \quad (22)$$

which constructs the complete orthogonal set together with states $\{|j, \pm\rangle, |j, 0\rangle\}$. Of particular interest, $|\psi_j\rangle$ is the eigenstate of H with energy $2\omega_a$. In the expression of Eq. (17), the zero-energy term represents this point, where we have ignored a constant shift $2\omega_a$. We will discuss this problem in next section.

Consequently, within a specific invariant subspace, the system made of N cavity array with a single two-level atoms embedded in each cavity appears to be equivalent to a tight-binding chain for spin-1 particle with spin-orbit interaction. The structures of H_{SO} is schematically illustrated in Fig. 1. Intuitively, the graph of H_{SO} consists of two unconnected subgraphs. This can be seen by observing that the parity operator

$$\hat{\Pi} = (-1)^{j+S_z+1} \quad (23)$$

with $\hat{\Pi} |j, S_z\rangle = \Pi |j, S_z\rangle$ and $\Pi = \pm 1$, characterizing the two subgraphs.

Then we conclude that the equivalent Hamiltonian H_{SO} can be decomposed into two independent parts

$$H_{\text{SO}} = H_o + H_e, \quad (24)$$

with $[H_o, H_e] = 0$, and $[\hat{\Pi}, H_e] = [\hat{\Pi}, H_o] = 0$. The sub-Hamiltonians are defined as

$$\begin{aligned} H_o = i\kappa \sum_{j=1,3,5,\dots} \underline{|j\rangle} S_x (1 - S_z^2) \underline{\langle j+1|} \\ + i\kappa \sum_{j=2,4,6,\dots} \underline{|j\rangle} S_x S_z^2 \underline{\langle j+1|} + \text{H.c.} \\ + 2\lambda \sum_{j=1,3,\dots} \underline{|j\rangle} S_z \underline{\langle j|}, \end{aligned} \quad (25)$$

and

$$\begin{aligned} H_e = \sqrt{2}i\kappa \underline{|0\rangle} S_x (1 - S_z^2) \underline{\langle 1|} \\ + i\kappa \sum_{j=2,4,6,\dots} \underline{|j\rangle} S_x (1 - S_z^2) \underline{\langle j+1|} \\ + i\kappa \sum_{j=1,3,5,\dots} \underline{|j\rangle} S_x S_z^2 \underline{\langle j+1|} + \text{H.c.} \\ + \sqrt{2}\lambda \underline{|0\rangle} S_z \underline{\langle 0|} + 2\lambda \sum_{j=2,4,6,\dots} \underline{|j\rangle} S_z \underline{\langle j|}. \end{aligned} \quad (26)$$

The subscripts o and e represent contributions associated with the sites with odd and even parity Π . The structures of H_o and H_e are schematically illustrated in Fig. 1. It indicates that the invariant space with $k = \pi$ is split in two unconnected subspaces. This allow us to investigate the Hamiltonians $H_{o,e}$ separately.

V. EXACT BOUND-PAIR STATES

Based on the above analysis, besides states $|\psi_j\rangle$, one can also construct a series of bound-pair states as the form

$$\begin{aligned} |\varphi_j\rangle = \frac{1}{\sqrt{\Omega_j}} [a_j (|j, +\rangle - |j, -\rangle) \\ + i2\sqrt{2}(\lambda/\kappa) |j+1, 0\rangle \\ - (|j+2, +\rangle - |j+2, -\rangle)], \end{aligned} \quad (27)$$

where the normalization factor Ω_j and amplitudes a_j

$$\Omega_j = 2(a_j)^2 + 8(\lambda/\kappa)^2 + 2, \quad (28)$$

and

$$a_j = \begin{cases} 2, & j = 0 \\ 1 & j \geq 1 \end{cases}. \quad (29)$$

Straightforward derivation shows that

$$\begin{aligned} H_e |\varphi_j\rangle &= 0, \text{ even } j \\ H_o |\varphi_j\rangle &= 0, \text{ odd } j \end{aligned} \quad (30)$$

i.e., $|\varphi_j\rangle$ is an eigenstate of H_{SO} . This is a direct application of the theorem about the bound state on the graph [16], which states that any eigenstate of a sub-graph is also the eigenstate of the whole, if the nodes cover all the joint points. We are interested in the expression of these states in the atom-photon basis. It is given by

$$\begin{aligned} |\varphi_j\rangle = \sum_l \frac{(-1)^l}{\sqrt{N\Omega_j}} [a_j (|1\rangle_l |1\rangle_{l+j} + |e\rangle_l |e\rangle_{l+j}) \\ - 2(\lambda/\kappa) (|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1}) \\ + (|1\rangle_l |1\rangle_{l+j+2} + |e\rangle_l |e\rangle_{l+j+2})] \end{aligned} \quad (31)$$

for $j \geq 1$, and

$$\begin{aligned} |\varphi_0\rangle = \sum_l \frac{(-1)^l}{\sqrt{N\Omega_0}} [a_0 \sqrt{2} |2\rangle_l \\ - 2(\lambda/\kappa) (|1\rangle_l |e\rangle_{l+1} - |e\rangle_l |1\rangle_{l+1}) \\ + (|1\rangle_l |1\rangle_{l+2} + |e\rangle_l |e\rangle_{l+2})]. \end{aligned} \quad (32)$$

Alternatively, direct derivation can check our conclusion for the original Hamiltonian of a lattice atom-photon system in Eq. (1) that

$$H |\varphi_j\rangle = 2\omega_a |\varphi_j\rangle. \quad (33)$$

The formation mechanism of these bound-pair eigenstates can be understood as the result of quantum interference in the following three different types of processes.

(i) We start with the case of switching off the JC interaction, $\lambda = 0$. The atoms are decoupled from the cavity array. It is readily to check that

$$[\eta_j, H - \omega_a \sum_l a_l^\dagger a_l] = 0, \quad (34)$$

where the operator η_j is defined as

$$\eta_j = \sum_l (-1)^l a_l^\dagger a_{l+j}^\dagger. \quad (35)$$

According to the similiar analysis in Ref. [17], it is found that state

$$|\Psi_n\rangle = (\eta_j)^n |G\rangle \quad (36)$$

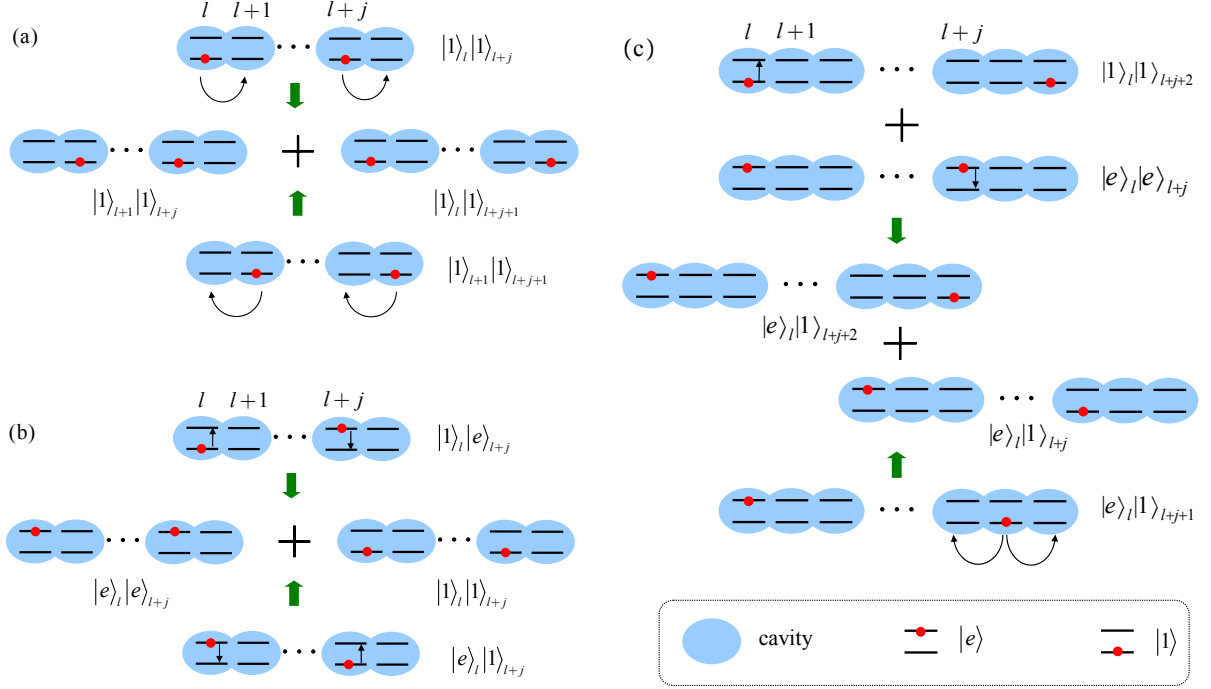


FIG. 2. (Color online) Schematic illustration for the mechanism of the formation of bound pair eigenstates. There are three types of destruction interference processes which result in the exact eigenstate $|\varphi_j\rangle$. (a) The Hubbard-type process represented in Eq. (40). (b) The JC-type process represented in Eq. (42). (c) The key process referred as mixed-type in Eq. (44), shows that the cancellation of the transitions requires the optimal ratio between the parameters λ and κ .

is an eigenstate of H ,

$$H |\Psi_n\rangle = 2n\omega_a |\Psi_n\rangle. \quad (37)$$

Furthermore, it is worth to note that even for a Bose Hubbard model, which involves the on-site interaction,

$$H_{\text{BH}} = -\kappa \sum_{l=1}^N \left(a_l^\dagger a_{l+1} + \text{H.c.} \right) + \frac{U}{2} a_l^\dagger a_l \left(a_l^\dagger a_l - 1 \right), \quad (38)$$

we still have

$$[\eta_j, H_{\text{BH}}] |G\rangle = 0, \quad (39)$$

which leads to the conclusion that $|\Psi_1\rangle$ is an eigenstate of H_{BH} .

The essence of the construction of $|\Psi_1\rangle$ is due to the destructive interference between the two transitions from states $|1\rangle_l |1\rangle_{l+j}$ and $|1\rangle_{l+1} |1\rangle_{l+j+1}$

$$\left. \begin{aligned} H |1\rangle_l |1\rangle_{l+j} \\ H |1\rangle_{l+1} |1\rangle_{l+j+1} \end{aligned} \right\} \longrightarrow |1\rangle_{l+1} |1\rangle_{l+j} + |1\rangle_l |1\rangle_{l+j+1}, \quad (40)$$

which results in

$$H \left(|1\rangle_l |1\rangle_{l+j} - |1\rangle_{l+1} |1\rangle_{l+j+1} \right) \longrightarrow 0. \quad (41)$$

Here the contribution of H_0 is ingored. We refer this as Hubbard-type process.

(ii) Now we consider the case of switching off the tunneling between cavities, $\kappa = 0$. Each cavity becomes separated from its neighbors. We have the identity

$$\left. \begin{aligned} H |e\rangle_l |n-1\rangle_l |n\rangle_{l+j} \\ H |n\rangle_l |e\rangle_{l+j} |n-1\rangle_{l+j} \end{aligned} \right\} = \lambda n |n\rangle_l |n\rangle_{l+j}, \quad (42)$$

which results in

$$H \left(|e\rangle_l |n-1\rangle_l |n\rangle_{l+j} - |n\rangle_l |e\rangle_{l+j} |n-1\rangle_{l+j} \right) = 0. \quad (43)$$

This means that there is a destructive interference between the two paths, which are the atom-photon transitions in the two different cavities l and $l+j$. It is a pure QED process in a JC model, which is referred as the JC-type process. It is easy to check that the combination of Hubbard and JC-type processes result in the formation the eigen state $|\psi_j\rangle$.

(iii) The crucial process that makes state $|\varphi_j\rangle$ become an eigenstate of the complete Hamiltonian is the combination of the above two. In this case, the excitation number must be 2. The transitions which result in the

destructive interference are

$$\left. \begin{aligned} & (1/\lambda) \left(|e\rangle_l |e\rangle_{l+j} + |1\rangle_l |1\rangle_{l+j+2} \right) \\ & (-1/\kappa) |e\rangle_l |1\rangle_{l+j+1} \end{aligned} \right\} \quad (44)$$

$$\rightarrow \left(|e\rangle_l |1\rangle_{l+j} + |e\rangle_l |1\rangle_{l+j+2} \right).$$

We note that the cancellation occurs only if the amplitudes of the two components $|e\rangle_l |e\rangle_{l+j} + |1\rangle_l |1\rangle_{l+j+2}$ and $|e\rangle_l |1\rangle_{l+j+1}$ are properly assigned. We refer this to the mixed-type process. In Fig. 2, three processes for the formation mechanism of the bound pair state is schematically illustrated.

VI. LONG-RANGE ENTANGLEMENT

We now study the feature of the obtained eigenstates. Apparently, the pair state $|\psi_j\rangle$ and $|\varphi_j\rangle$ are entangled states. In the strong coupling limit $\lambda \gg \kappa$, we have

$$|\varphi_j\rangle \approx \sum_l \frac{(-1)^l}{\sqrt{2N}} \left(|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1} \right), \quad (45)$$

which is the superposition of entangled states between two cavities at distance $j+1$. States

$$(|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1})/\sqrt{2} \quad (46)$$

in $|\varphi_j\rangle$ and

$$(|e\rangle_l |e\rangle_{l+j} - |1\rangle_l |1\rangle_{l+j})/\sqrt{2} \quad (47)$$

in $|\psi_j\rangle$ are both maximally entangled states of l -th and $(l+j)$ -th (or $(l+j+1)$ -th) cavities for the two modes, excited cavity field and excited atom modes. To demonstrate this concept in a precise manner, we introduce lower branch and upper branch exciton-polariton states,

$$|\downarrow\rangle_l = \frac{1}{\sqrt{2}}(i|1\rangle_l - |e\rangle_l), \quad (48)$$

$$|\uparrow\rangle_l = \frac{1}{\sqrt{2}}(i|1\rangle_l + |e\rangle_l), \quad (49)$$

the superposition of which yields a polariton qubit state at cavity l . With $|\downarrow\rangle_l$ and $|\uparrow\rangle_l$ being basis, it is given that

$$|\varphi_j\rangle \sim \frac{1}{\sqrt{2}}(|\uparrow\rangle_l |\uparrow\rangle_{l+j+1} - |\downarrow\rangle_l |\downarrow\rangle_{l+j+1}), \quad (50)$$

$$|\psi_j\rangle \sim \frac{1}{\sqrt{2}}(|\uparrow\rangle_l |\uparrow\rangle_{l+j} + |\downarrow\rangle_l |\downarrow\rangle_{l+j}), \quad (51)$$

which are standard Bell states. We note that the entanglement does not decrease as the distance j increases.

The entanglement is one of the great importance for the new field of quantum information theory. Polaritons [18] as quasiparticles of light and matter, are the most promising solution for the interface between electronic and photonic qubit states.

However, we would like to point out that two atoms for state $|\psi_j\rangle$ (or $|\varphi_j\rangle$), in l -th and $(l+j)$ -th (or $(l+j+1)$ -th) cavities, do not entangle with each other due to the following reason. The atomic entanglement can be characterized by concurrence [6]. The reduced density matrix for two atoms in l -th and $(l+j+1)$ -th cavities is

$$\rho^{(l,l+j+1)} = \text{Tr}_p \text{Tr}_{(l,l+j+1)} (|\varphi_j\rangle \langle \varphi_j|), \quad (52)$$

where Tr_p denotes the trace over all photon variables and $\text{Tr}_{(l,l+j+1)}$ denotes the trace over all atomic variables except for l -th and $(l+j+1)$ -th atoms. It has been shown in Refs. [6] that the formula for the concurrence of two quasi-spin in a hybrid system is the same as that for pure spin-1/2 system [19, 20]. Then the concurrence $C_{ll'}$ shared between two atoms l and l' is obtained as

$$C_{ll'} = 2 \max(0, |z_{ll'}| - \sqrt{u_{ll'}^+ u_{ll'}^-}). \quad (53)$$

in terms of the quantum correlations

$$z_{ll'} = \langle \varphi_j | \sigma_l^+ \sigma_{l'}^- | \varphi_j \rangle, \quad (54)$$

$$u_{ll'}^\pm = \frac{1}{4} \langle \varphi_j | (1 \pm \sigma_l^z) (1 \pm \sigma_{l'}^z) | \varphi_j \rangle, \quad (55)$$

where $\sigma_i^+ = (\sigma_i^-)^\dagger = |e\rangle_i \langle g|$. It is a straightforward calculation to show that $C_{ll'}$ is always zero for both states $|\varphi_j\rangle$ and $|\psi_j\rangle$.

VII. SUMMARY

In summary, we have established the link between the two-excitation JCH model and the single-particle 4-leg ladder with an effective flux, which is shown to be equivalent to a chain system of spin-1 particle with spin-orbit coupling. It also introduces a mechanism to construct a series of bound-pair eigenstates, which display long-range polaritonic entanglement. This finding reveals that the hybrid system can offer rich features and useful functionality, which will motivate further investigation.

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